Eur. Phys. J. C 16, 677–681 (2000)
Digital Object Identifier (DOI) 10.1007/s100520000445

THE EUROPEAN
PHYSICAL JOURNAL C

Vacuum structure of a modified MIT bag

R. Hofmann[1], M. Schumann[2,3], T. Gutsche[1], R.D. Viollier[3]

[1] Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, 72076 Tübingen, Germany
[2] Institut für Theoretische Physik, Universität Basel, Klingelbergstrasse 82, 4056 Basel, Switzerland
[3] Institute of Theoretical Physics and Astrophysics, Department of Physics, University of Cape Town, Rondebosch 7701, South Africa

Received: 4 April 2000 / Published online: 6 July 2000 – © Springer-Verlag 2000

Abstract. An alternative to introducing and subsequently renormalizing classical parameters in the expression for the vacuum energy of the MIT bag for quarks is proposed in the massless case by appealing to the QCD trace anomaly and scale separation due to asymptotic freedom. The explicit inclusion of gluons implies an unrealistically low separation scale.

1 Introduction

The vacuum energies of spatially confined quantum fields have been of great interest since the early days of quantum field theory [1,2]. Shortly after the advent of the non-Abelian gauge theory of strong interactions [3–5], the bag models of hadrons [6–8] required estimates for the contribution of the spherically constrained vacuum to the total energy of a hadron. In essence, two lines of approaches have been pursued in the past. The canonical vacuum energy was parametrized by means of a dimensionless quantity Z_0 to be fitted to experiment [9]. While disregarding the quadratic boundary condition of the original MIT bag model, a relation between the bag radius R and the bag constant B was established by demanding stability of the calculated hadron mass under variations of R [10]. However, the quadratic boundary condition of the fermionic MIT bag model, $B_q = -\frac{1}{2}\partial_r \left(\bar{\psi}\psi\right)\big|_{r=R}$, was introduced to restore the broken four-momentum conservation of the bag [6], and thus it should be taken seriously. For a meaningful definition of the bag constant B_q according to the quadratic boundary condition, the vacuum expectation value of this operator equation must be taken [11].

There has been a great effort to *compute* the Casimir effect of the MIT bag model [11–16]. The vacuum expectation values of global quantities must be regularized. Several procedures, adapted to either global or local approaches, were applied. Global techniques regularize the sum over mode energies by analytical continuation (zeta-function method) [13,14,17], while local approaches compute finite densities based on two-point functions. The space-integral of these densities is regularized by volume or temporal cutoffs [2,18]. However, different regularization schemes yield different answers which is not acceptable. Various solutions have been suggested [11,13–15]. For instance, the vacuum energy has been separated into a classical and a quantum part. The classical contribution was parametrized by phenomenological quantities to absorb divergences due to the quantum part by appropriate renormalizations [13,15]. This procedure relies on direct experimental information which is unsatisfactory. Interesting results were obtained in the massive case [13,14,19]. By imposing the condition that the vacuum of a very massive field should not fluctuate, a unique term in the canonical vacuum energy, attributed to quantum fluctuations, was isolated.

In this paper we propose an alternative to the above procedure. Our approach is based on a separation between the perturbative and nonperturbative regimes of QCD. As suggested by Vepstas and Jackson in the framework of a chiral bag model [20], hard fluctuations should be allowed to traverse the boundary since these fluctuations are not subject to the low-energy confinement mechanism. In contrast to the work of [20], we consider only the interior of the bag. In the simple model of the QCD vacuum, which the bag-model philosophy offers, we think of hard fluctuations to be noninteracting and unconfined when calculating nonperturbative effects, such as the ground state energy of the bag.

2 Calculation

Our numerical method to compute the regularized canonical vacuum energy and the bag constant of the fermionic MIT bag was explained in [21]. The procedure is based on a mode sum representation of the cavity propagator. A Schwinger parametrization of the Euclidean “momentum-squared” denominator and a subsequent integration over the “off-shell” parameter ω are performed.

Correspondence to: Marc.Schumann@unibas.ch

2.1 The canonical vacuum energy density

Under the condition, that the free-space vacuum energy vanishes, we obtain the angular integrated form of the canonical vacuum energy density $\left\langle\tilde{\theta}^{00}\right\rangle$ as

$$\begin{aligned}\left\langle\tilde{\theta}^{00}(r)\right\rangle &\equiv 4\pi\ \left\langle\theta^{00}(r)\right\rangle \qquad (1)\\ &= \frac{1}{2\ \pi^{1/2}}\int_{1/\lambda^2}^{\infty} dz\ \frac{1}{z^{3/2}}\\ &\times\Big[\sum_{\kappa}\frac{1}{2}\sum_{n}^{n_\lambda}\frac{1}{R^3}\,\mathcal{N}^2_{n,\kappa}\ (2J+1)\\ &\qquad \left((j_l(|\varepsilon_{n,\kappa}|r))^2+(j_{\bar{l}}(|\varepsilon_{n,\kappa}|r))^2\right)\ \mathrm{e}^{-z\varepsilon^2_{n,\kappa}}\\ &-\sum_{l}\ \frac{4}{\pi}\ (2l+1)\int_0^{\lambda} dk\ k^2(j_l(kr))^2\ \mathrm{e}^{-zk^2}\Big]\,,\\ J&=|\kappa|-\frac{1}{2}\,,\quad l=|J|+\frac{1}{2}\ \mathrm{sgn}\ \kappa\,,\quad \bar{l}=l-\mathrm{sgn}\ \kappa\,.\end{aligned}$$

Thereby, j_l denotes the spherical Bessel function, and the subscripts n, κ, and μ stand for the radial quantum number, the Dirac quantum number, and the angular momentum projection, respectively. The radial quantum number n_λ labels the mode energy closest to λ, and $\mathcal{N}^2_{n,\kappa}$ is a normalization constant (see [21]). In (1) the integral over k corresponds to the free-space subtraction. Hard fluctuations are excluded by distinguishing two cases: 1) hard fluctuations with $\omega,\ \varepsilon_{n,\kappa}>\lambda$ or $\omega\leq\lambda,\ \varepsilon_{n,\kappa}>\lambda$ are omitted by truncation of the mode sum, and 2) hard fluctuations with $\omega>\lambda$, $\varepsilon_{n,\kappa}\leq\lambda$ are discarded by restriction of the z-integration. The canonical vacuum energy E is given by $E=\int_0^R \mathrm{d}r\ r^2\ \left\langle\tilde{\theta}^{00}(r)\right\rangle$.

2.2 The fermionic bag constant

Due to the vacuum expectation value of the quadratic boundary condition, the fermionic bag constant B_q reads

$$\begin{aligned}B_q &= -\frac{1}{4\pi^{3/2}}\int_{1/\lambda^2}^{\infty} dz\,\frac{1}{z^{1/2}}\sum_{\kappa}(2J+1)\\ &\times\sum_{n>0}^{n_\lambda}\frac{1}{R^3}\mathcal{N}^2_{n,\kappa}\,\varepsilon^2_{n,\kappa}\mathrm{e}^{-z\varepsilon^2_{n,\kappa}}\\ &\times\Big[\frac{j_l(|\varepsilon_{n,\kappa}|R)}{2l+1}\ (l\ j_{l-1}(|\varepsilon_{n,\kappa}|R)\\ &\qquad -(l+1)\,j_{l+1}(|\varepsilon_{n,\kappa}|R))\\ &-\frac{j_{\bar{l}}(|\varepsilon_{n,\kappa}|R)}{2\bar{l}+1}\ \left(\bar{l}\,j_{\bar{l}-1}(|\varepsilon_{n,\kappa}|R)\right.\\ &\qquad\left.-(\bar{l}+1)j_{\bar{l}+1}(|\varepsilon_{n,\kappa}|R)\right)\Big]\,. \qquad (2)\end{aligned}$$

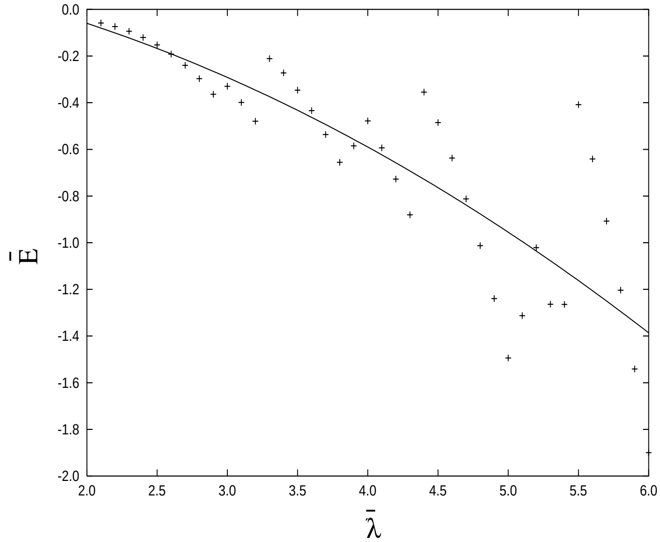

Fig. 1. The canonical part of the one-flavor, one-color vacuum energy in dependence on the cutoff. Both quantities are given in units of R^{-1}

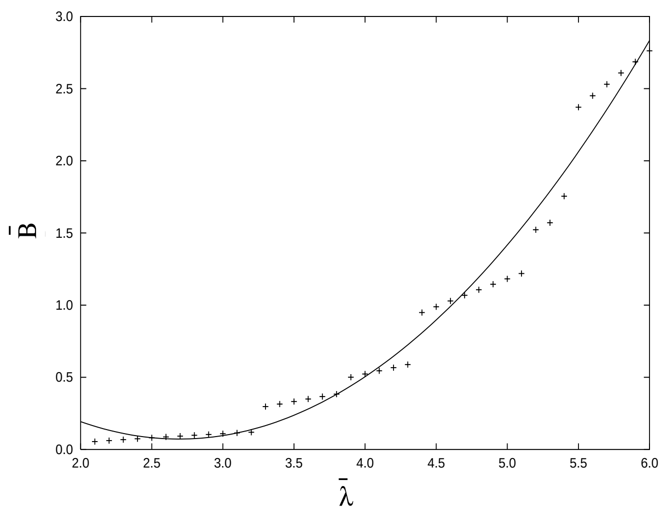

Fig. 2. The one-flavor, one-color fermionic bag constant in dependence on the cutoff. The bag constant and the cutoff are given in units of R^{-4} and R^{-1}, respectively

Figure 1 shows the result of the calculation of $\bar{E}\equiv R\times E$ as a function of $\bar{\lambda}\equiv R\times\lambda$. The discontinuous behavior is due to the fact that mode eigenvalues at low energies are not spaced equidistantly. To smooth the "nervous" behavior, we use a quadratic regression as indicated by the solid line. In Fig. 2 the $\bar{\lambda}$ dependence of $\bar{B}_q\equiv R^4\times B_q$ is depicted. Again, a quadratic fit is used to average over discontinuities. Tables 1 and 2 contain a list of values for $3\times n_f\times B_q$, $-3\times n_f\times E$ under variation of R, where $\bar{\lambda}$ is adjusted to $\lambda=1.2$ GeV, $\lambda=1.6$ GeV and $\lambda=0.8$ GeV, $\lambda=1.0$ GeV, respectively. Thereby, $n_f=2$ stands for the light-flavor multiplicity, and the factor three is the number of colors.

Table 1. The dependence of the fermionic bag constant and the canonical part of the fermionic vacuum energy on the cutoff $\bar{\lambda} = \lambda \times R$ for two light-quark flavors with R ranging from 0.4 fm to 1.0 fm. The lower and upper values of $\bar{\lambda}$ correspond to $\lambda = 1.2$ GeV and $\lambda = 1.6$ GeV, respectively

R [fm]	0.4		0.5		0.6		0.7		0.8		0.9		1.0	
$\bar{\lambda}$	2.4	3.2	3.0	4.1	3.6	4.9	4.3	5.7	4.9	6.5	5.5	7.3	6.1	8.1
$3 \times n_f \times B_q$[GeV^4]	0.032	0.053	0.024	0.079	0.021	0.089	0.026	0.088	0.028	0.082	0.028	0.075	0.027	0.068
$-3 \times n_f \times E$ [GeV]	0.450	1.060	0.716	1.439	0.942	1.779	1.145	2.095	1.334	2.398	1.513	2.690	1.686	2.976

Table 2. Same as in Table 1. The lower and upper values of $\bar{\lambda}$ correspond to $\lambda = 0.8$ GeV and $\lambda = 1.0$ GeV, respectively

R [fm]	0.4		0.5		0.6		0.7		0.8		0.9		1.0	
$\bar{\lambda}$	1.6	2.0	2.0	2.5	2.4	3.0	2.8	3.5	3.2	4.1	3.6	4.6	4.1	5.1
$3 \times n_f \times B_q$ [GeV^4]	0.129	0.065	0.027	0.011	0.006	0.007	0.003	0.010	0.003	0.012	0.004	0.013	0.005	0.014
$-3 \times n_f \times E$ [GeV]	−0.031	0.193	0.154	0.415	0.300	0.597	0.422	0.755	0.530	0.900	0.628	1.034	0.720	1.162

Appealing to the one-loop trace-anomaly [22] of the QCD energy-momentum tensor $\theta^{\mu\nu}$

$$\langle \theta^\mu_\mu \rangle = -\frac{1}{8}\left(11 - n_f \frac{2}{3}\right)\left\langle \frac{\alpha_s}{\pi} F^a_{\kappa\nu} F^{\kappa\nu}_a \right\rangle \,, \tag{3}$$

we assume for the moment that only quark fluctuations contribute to the bag constant. Using the fact that the canonical part of $\theta^{\mu\nu}$ is traceless in the mixed MIT bag model, we obtain (apart from a sign) the relation

$$3 \times n_f \times B_q = 0.302 \times \left\langle \frac{\alpha_s}{\pi} F^a_{\kappa\nu} F^{\kappa\nu}_a \right\rangle \,. \tag{4}$$

Thereby, the value of the (renormalization-scale independent) gluon condensate [23] is $\left\langle \frac{\alpha_s}{\pi} F^a_{\mu\nu} F^{\mu\nu}_a \right\rangle = 0.024 \pm 0.012$ GeV^4. Comparing by means of (4) the central value of the gluon condensate with the values of $3 \times n_f \times B_q$ (Tables 1, 2), which are stable under variation of R, we obtain agreement for $\lambda = 1.0$ GeV and a bag radius R of 0.6 fm. Given these values of λ and R, the results of Table 2 indicate that $-3 \times n_f \times E$ is close to phenomenologically obtained values: In [9] Z_0 parametrizes the Casimir energy as $-Z_0/R$. Fits to the hadron spectrum yield values of about $Z_0 = 2$ [9]. The effect of the center-of-mass contribution to Z_0 was found to be of the order of 40% in [24, 25]. In comparison, our value of $-3 \times n_f \times E = 0.597$ GeV at $R = 0.6$ fm corresponds to Z_0=1.79 with no center-of-mass contribution.

2.3 The gluonic bag constant

How do confined gluons alter the results obtained so far? Analogous to the fermionic case the gluonic bag constant $8 \times B_g$ is defined as the vacuum expectation value of the following quadratic boundary condition [6]

$$B_g = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) \,, \tag{5}$$

where to lowest order in the coupling the field strength tensor $F_{\mu\nu}$ is Abelian, and $\mathbf{E}$ and $\mathbf{B}$ denote the electric and magnetic field strength, respectively. Appealing in the sourceless case to the symmetry of Maxwell's equations under the duality transformation $\mathbf{E} = \mathbf{B}^D$, $\mathbf{B} = -\mathbf{E}^D$, we obtain due to physical transverse polarizations (TE,TM) the following expression for B_g in Feynman gauge

$$\begin{aligned}
B_g = \frac{1}{32\pi^{3/2}} \frac{1}{R^3} \int_{1/\lambda^2}^{\infty} dz\, \frac{1}{z^{3/2}} \\
\times \sum_{n,J\geq 1} \Big\{ (2J+1) \Big[(\mathcal{N}^{\mathrm{TM}}_{n,J})^2 j_J^2(\varepsilon^{\mathrm{TM}}_{n,J} R) \mathrm{e}^{-z(\varepsilon^{\mathrm{TM}}_{n,J})^2} \\
- (\mathcal{N}^{\mathrm{TM},D}_{n,J})^2 j_J^2(\varepsilon^{\mathrm{TM},D}_{n,J} R) \mathrm{e}^{-z(\varepsilon^{\mathrm{TM},D}_{n,J})^2} \Big] \\
+ (\mathcal{N}^{\mathrm{TE}}_{n,J})^2\, \mathrm{e}^{-z(\varepsilon^{\mathrm{TE}}_{n,J})^2} \\
\times \left[(J+1) j_{J-1}^2(\varepsilon^{\mathrm{TE}}_{n,J} R) + J\, j_{J+1}^2(\varepsilon^{\mathrm{TE}}_{n,J} R) \right] \\
- (\mathcal{N}^{\mathrm{TE},D}_{n,J})^2\, \mathrm{e}^{-z(\varepsilon^{\mathrm{TE},D}_{n,J})^2} \\
\times \left[(J+1) j_{J-1}^2(\varepsilon^{\mathrm{TE},D}_{n,J} R) + J\, j_{J+1}^2(\varepsilon^{\mathrm{TE},D}_{n,J} R) \right] \Big\}.
\end{aligned} \tag{6}$$

Thereby, the superscript D indicates that the corresponding eigenvalue has been obtained from the linear boundary condition $n_\mu (F^D)^{\mu\nu} = 0$ for the *dual* field strength, and $\mathcal{N}^{\mathrm{TE}}_{n,J}$ ($\mathcal{N}^{\mathrm{TM}}_{n,J}$) denotes the normalization constant for the corresponding mode. For technicalities concerning Cavity QCD in Feynman gauge see Refs. [26,27]. In (6), the introduction of the Schwinger parameter z and the subsequent truncation of the z-integration and mode summation due to the subtraction of hard fluctuations in the vacuum is analogous to the fermionic case. Table 3 contains the values for $8 \times B_g$ under variations of R with λ adjusted to $\lambda = 0.8$ GeV and $\lambda = 1.0$ GeV. For radii R less than $R = 0.7$ fm there is no contribution from the mode sum of (6). We find stability for $8 \times B_g$ under a variation of R at $R = 0.8$ fm with $8 \times B_g = 0.0128$ GeV^4 for $\lambda = 0.8$ GeV and with $8 \times B_g = 0.0189$ GeV^4 for $\lambda = 1.0$ GeV. Appealing to the QCD trace anomaly and requiring that the total bag constant $B \equiv 3 \times n_f \times B_q + 8 \times B_g$ produces

Table 3. The dependence of the gluonic bag constant on the cutoff $\bar{\lambda} = \lambda \times R$ with R ranging from 0.4 fm to 1.0 fm. The lower and upper values of $\bar{\lambda}$ correspond to $\lambda = 0.8$ GeV and $\lambda = 1.0$ GeV, respectively

R [fm]	0.4		0.5		0.6		0.7		0.8		0.9		1.0	
$\bar{\lambda}$	1.6	1.8	2.0	2.25	2.4	2.7	2.8	3.15	3.2	3.6	3.6	4.05	4.0	4.5
$8 \times B_g$ [GeV^4]	0	0	0	0	0	0	0.0133	0.0205	0.0128	0.0189	0.0179	0.0302	0.0191	0.0271

the central value of the gluon condensate, implies λ to be less than $\lambda = 0.8$ GeV. As far as the properties of the lowest light-flavor resonances are concerned, which are believed to be strongly correlated with the QCD condensates of lowest mass-dimension, QCD sum rules [28] suggest the onset of the perturbative regime at values of about 1.5–1.8 GeV^2 of the spectral continuum threshold s_0 [23,29–31]. This corresponds to λ=1.22–1.34 GeV. Hence, our value of $\lambda \approx 1.0$ GeV for the pure quark bag seems already a bit too small which might be due to the mode sum representation of the cavity propagator with implicit spatial correlations, whereas s_0 relates to plane-wave states. Nevertheless, it is hard to accept values of λ lower than 0.8 GeV for the mixed bag.

2.4 The deconfinement phase transition

In the standard fashion [32,33] we now estimate the critical temperature T_c (no baryonic chemical potential μ) of a deconfinement phase transition from the bag constant $3 \times n_f \times B_q$. For this we take the value $n_f \times B = 0.007$ GeV^4 for $\lambda = 1.0$ GeV, and with

$$4B \stackrel{!}{=} \pi^2 T_c^4 \left(\frac{8}{15} + \frac{7}{10} \right) + B \quad (7)$$

we obtain $T_c = 203.8$ MeV. From SU(3) Yang-Mills lattice simulations one expects a smooth decrease of the gluon condensate for temperatures near 260 MeV [34]. Therefore, we would have to correct the bag radius at zero temperature towards higher values near the phase transition. For comparison, we determine the critical temperature from the phenomenological value $B = 4.54 \times 10^{-4}$ GeV^4 of [9] as $T_c = 102.8$ MeV. This is too low, since otherwise the deconfinement phase transition would have already been seen experimentally [35].

3 Conclusion

In summary, invoking asymptotic freedom and appealing to the QCD trace-anomaly, the linear *and* nonlinear boundary condition of the MIT bag model for quarks provide a reasonable agreement of the calculated canonical vacuum energy with that found in hadron phenomenology which makes the introduction of phenomenological parameters redundant. However, the explicit inclusion of gluons drives the separation scale down to values which are not acceptable.

Acknowledgements. We thank K. Kirsten for a stimulating correspondence. Financial contributions from the Foundation for Fundamental Research (M.S. and R.D.V.), the Deutsches Bundesministerium fuer Bildung und Forschung, contract No. 06 Tue 887, (T.G.), and the Graduiertenkolleg "Struktur und Wechselwirkung von Hadronen und Kernen" (R.H.), are gratefully acknowledged.

References

1. H. B. G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948).
2. G. Plunien, B. Müller, and W. Greiner, Phys. Rep. **134**, 89 (1986).
3. H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. B **47**, 365 (1973).
4. D. J. Gross and F. Wilczek, Phys. Rev. **D 8**, 3633 (1973).
5. H. D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973).
6. A. Chodos *et al.*, Phys. Rev. **D 9**, 3471 (1974).
7. T. D. Lee, Phys. Rev. **D 19**, 1802 (1979).
8. W. A. Bardeen *et al.*, Phys. Rev. **D 11**, 1094 (1975).
9. T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. **D 12**, 2060 (1975).
10. A. Thomas, Adv. Nucl. Phys. **13**, 1 (1984).
11. K. A. Milton, Phys. Rev. **D 22**, 1444 (1980).
12. K. A. Milton, L. DeRaad, and J. Schwinger, Ann. Phys. **115**, 388 (1978).
13. E. Elizalde, M. Bordag, and K. Kirsten, J. Phys. A **31**, 1743 (1998).
14. M. Bordag, E. Elizalde, K. Kirsten, and S. Leseduarte, Phys. Rev. **D 56**, 4896 (1997).
15. M. Bordag, D. Robaschik, and E. Wieczorek, Ann. Phys. **165**, 192 (1985).
16. D. Deutsch and P. Candelas, Phys. Rev. **D 20**, 3063 (1979).
17. V. V. Nesterenko and I. G. Pirozhenko, Phys. Rev. **D 57**, 1284 (1998).
18. C. M. Bender and P. Hays, Phys. Rev. **D 14**, 2622 (1976).
19. K. Kirsten, private communication.
20. L. Vepstas and A. D. Jackson, Nucl. Phys. A **481**, 668 (1988).
21. R. Hofmann, M. Schumann, and R. D. Viollier, Eur. Phys. J. C **11**, 153 (1999).
22. T. Hatsuda, Y. Koike, and S. H. Lee, Nucl. Phys. B **394**, 221 (1993).
23. H. G. Dosch, Prog. Part. Nucl. Phys. **33**, 121 (1994).
24. J. F. Donoghue and K. Johnson, Phys. Rev. **D 21**, 1975 (1980).
25. K. A. Milton, Phys. Rev. **D 22**, 1441 (1980).
26. T. H. Hansson and R. L. Jaffe, Phys. Rev. **D 28**, 882 (1983).
27. R. F. Buser, R. D. Viollier, and P. Zimak, Int. J. Theo. Phys. **8**, 925 (1988).

28. M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B **147**, 385 (1979).
29. S. Narison, *QCD Spectral Sum Rules* (World Scientific, London, 1989).
30. H. G. Dosch and S. Narison, Phys. Lett. B **203**, 155 (1988).
31. C. A. Dominguez and E. de Rafael, Ann. Phys. **174**, 372 (1987).
32. J. Cleymans, R. V. Gavai, and E. Suhonen, Phys. Rev. **130**, 217 (1986).
33. U. Mosel, *Fields, Symmetries, and Quarks* (McGraw-Hill, New York, 1989).
34. D. E. Miller, Acta Phys. Polon. B **28**, 1937 (1997).
35. P. Braun-Munzinger, Nucl. Phys. A **663&664**, 183 (2000).